FUNDAMENTAL THEOREM OF PRIME NUMBERS

dal libro di Filippo Giordano *La ragione dei primi*, in vetrina sul sito internet www.ilmiolibro.it

(Traduzione di Vito Portera)

Enunciation:

Let's assume n as any natural and positive number. For any squared number N is always given a prime number within the numeric set included between $\underline{n^2 - n}$ and n^2 plus another prime number included in the numeric set included between n^2 and $\underline{n^2 + n}$ such as between any square *n* and the subsequent square *n* always exist at least two prime numbers.

Proposition

Let's assume *iT* (insieme Triangolare = Triangular set) as a set of natural positive consecutive numbers that starting from 1 reaches any number n-1: (1, 2, 3, … n-1).

Let's assume *iMa* (insieme multiplo, a = multiple set, a) as the set of numbers included between $n(n-1)$ and n^2, extremes excluded, and iMb (insieme multiplo, b = Multiple set, b) the set of elements included between n^2 and $n(n+1)$, extremes excluded.

Property 1

Any value attributed to *n* the number of the elements of the *iMa* is always equal to the number of the elements of the *iMb* and each of them, in its turn, has a number of elements equal to those of *iT*.

Example 1:

Let's assume n = 3,

the elements of *iT* are, by definition, 1, 2 (=2);

the elements of *iMa*, included between 3x2=6 and 3x3= 9, extremes excluded, are 7, 8 (=2);

the elements of *iMb*, included between 3x3=9 and 3x4=12, extremes excluded, are 10,11 (=2).

Example 2:

let's assume n = 4,

the elements of *iT* by definition, are 1, 2, 3 (=3);

the elements of *iMa*, included between 4x3 = 12 and 4 x 4= 16, extremes excluded, are: 13, 14, 15 (=3);

the element of *iMb*, included between 4x4=16 and 4x5=20, extremes ecluded, are = 17, 18, 19 (=3).

Lemma 1

The natural order of whole positive numbers is alternate by odd and even numbers that go on to infinity; starting from 1, for each couple of consecutive elements occurs a parity off odd and even numbers; for each odd quantity of consecutive elements, odd elements are prevailing in the set.

Property 2

Since the *iT* is defined by a number of *n -1* elements, *iT* has a number of even elements if n is odd while it has a number of odd elements if *n* is even. Since both numbers of *iMa* and iMb are equal to n-1, both such sets will have a number of odd elements if *n* is even while they have a number of even elements if *n* is odd.

Example 3:
Let's assume n=5,
the elements of *iT* are 1, 2, 3, 4 (= 4 elements)
the elements of *iMa*, extremes excluded, are: 21, 22, 23, 24 (= 4 elements);
the elements of *iMb*, extremes excluded, are: 26, 27, 28, 29, (= 4 elements).

Example 4:
Let's assume n = 6,
the elements of *iT* are 1, 2, 3, 4, 5 (= 5 elements)
the elements of *iMa*, extremes excluded, are: 31, 32, 33, 34, 35 (= 5 elements);
the elements of *iMb*, extremes excluded, are: 37, 38, 39, 40, 41 (= 5 elements).

Property 3

Within the symmetry of the comprehensive number of elements of each *iT*, *iMa* and *iMb* also the number of even and odd numbers is symmetrical in each set. In fact, if n is odd the set of the elements is even and then the number of odd elements species is equal to the number of even elements species; on the contrary, when *n* is even the number of elements of each set is odd, and odd elements are prevaling.

<u>Example 5</u>
Let's assume $n = 7$:
the elements of *iT* are: 1, 2, 3, 4, 5, 6; total 6, being three odd and three even;
the elements of *iMa* are: 43, 44, 45, 46, 47, 48; total 6, being three odd and three even;
the elements of *iMb* are: 50, 51, 52, 53, 54, 55; total 6, being three odd and three even.

<u>Example 6</u>
Let's assume $n = 8$:
the elements of *iT* are: 1, 2, 3, 4, 5, 6, 7; total 7, being four odd and three even;
the elements of *iMa* are: 57, 58, 59, 60, 61, 62, 63; total 7, being four odd and three even;
the elements of *iMb* are: 65, 66, 67, 68, 69, 70, 71; total 7, being four odd and three even.

Property 4

a) The sum of the elements of *iT* is given by the Gauss formula, (opportunely modified since it doesn't contain n): $\boldsymbol{n(n-1)/2}$.
b) The sum of elements of *iMa* is equal to the result of $\boldsymbol{n(n-1)^2 + n(n-1)/2}$ formula in wich the former part $\boldsymbol{[n(n-1)^2]}$ is a multiple of the sum of the elements of *iT* e and the latter part $\boldsymbol{[n(n-1)/2]}$ *is equal to iT*.
c) The sum of the elements of *iMb* is equal to the result of $\mathbf{(n-1)n^2 + n(n-1)/2}$ formula, in wich the former part $\boldsymbol{[(n-1)n^2]}$ is a multiple of the sum of the elements of *iT* and the latter part $\boldsymbol{[n(n-1)/2]}$ is equal to *iT*.

<u>Example 7</u>
Let's assume $n = 9$,
Sum of the elements $\boldsymbol{iT = n(n-1)/2} = 9 \times 8 : 2 = 36$;

Sum of the elements ***iMa*** = $\boldsymbol{n(n-1)^2 + n(n-1)/2}$ = (9 x 8 x 8) + (9 x 8:2) = 576 + 36 = 612;
Sum of the elements ***iMb*** = $\boldsymbol{(n-1)n^2 + n(n-1)/2}$ = (8 x 9 x 9) + (9 x 8:2) = 648 + 36 = 684;

<u>Example 8</u>
Let's assume n = 10,
Sum of the elements ***iT*** = **n(n-1)/2** = 10 x 9 : 2 = 45;
Sum of the elements ***iMa*** = $\mathbf{n(n-1)^2 + n(n-1)/2}$ = (10 x 9 x 9) + (10 x 9:2) = 810 + 45 = 855;
Sum of the elements ***iMb*** = $\mathbf{(n-1)n^2 + n(n-1)/2}$ = (9 x 10 x 10) + (10 x 9:2) = 900 + 45 = 945;

Passing note

The formula identifies both in the *iMa* and in the *iMb* a part (the former) common to all elements (as it were the surname of brothers belonging to the same family group – a comparison mathematically unfit but suitable to render the idea of belonging).
In order to carry out the formula I imagined each one of the elements *iMa* and *iMb* as sustained by a common basis. What is such basis?
Let's consider $n = 9$.
The eight elements *iMa* are the numbers 73, 74, 75, 76, 77, 78, 79, 80. Their common basis is equal to the value of the smallest element -1, that is $n^2(n-1)$ = 9x8 = 72. Such value, multiplied by the number of the elements, gives the comprensive sum of the basis 72x8 = 576.
To the comprehensive value of the common basis must be added the value of the difference of each element with the basis.

Thus we have:
72 + 1 = 73;
72 + 2 = 74;
72 + 3 = 75;
72 + 4 = 76;
72 + 5 = 77;
72 + 6 = 78;

72 + 7 = 79;
72 + 8 = 80;
We can easily note that the value of each of such differences follows, step by step, the value of the pertinent elements *iT* : 1, 2, 3, 4, 5, 6, 7, 8 (as it were a name given at baptism) In order to quantify the comprehensive value of the iMa differences from the basis value we have but to adopt the same formula used to quantify the sum of *iT* elements. The same process is used with the iMb basis wich, instead, is given by the value of n^2, since all its elements are successive to n^2. In the end we have $\underline{81 \times 8} + \underline{8 \times 9 / 2}$.
Note that:

Property 5

For the same value of n, both the sum of the bases value and the sum of the differences (iMa and iMb) is always divisible by the sum of the value of *iT* elements.

<u>Example 9</u>
Let's assume n = 9,
the sum of the *iMa* elements', value equal to 576, divided by the sum of the *iT* elements' values, equal to 36, is 16, whereas the sum of the differences between bases and values of the elements iMa (36), results identical to the sum of the elements iT = 36; then 36:36 = 1 whereas, for *iMb* we have 648:36 =18; 36:36 = 1.
Furthermore, in particolar,

Property 6

a) The sum of the elements' values of *iMa* is always divisible by the sum of the elements' value *iT* and the result is given by the formula 2n-1.
b) The sum of the elements' values of *iMb* is always divisible by the sum of the elements' values iT and the result is given by the formula 2n +1.

<u>Example 10</u>
Let's assume = 9,
2n-1 = 17;
2n+1 = 19;
iT = 36;
iMa = 612;
iMb = 684;
then:
(iMa) 612 : (iT) 36 = 17 = 2n-1;

(iMb) 684 : (iT) 36 = 19 = 2n+1;

Example 11

Let's assume *n* = 10,

2n-1 = 19;

2n+1 = 21;

iT = 45;

iMa = 855;

iMb = 945;

then:

(*iMa*) 855 : (*iT*) 45 = 19 = 2n-1;

(*iMb*) 945 : (*iT*) 45 = 21 = 2n+1.

Property 7

For the same value of n, the sum of odd elements' values of *iMa* and *iMb* is always divisible by the sum of the odd elements' values *iT*.

Example 12:

Let's assume n = 15

The sum of the odd elements' values *iMa* (211 + 213 + 215 + 217 + 219 + 221 + 223 = 1519) divided by the sum of the elements' corresponding values *iT* (1+ 3+ 5+ 7 + 9+ 11+ 13 = 49) is the whole number 31, in its turn equal to 2n+1

Lemma 2

For any natural number *n* factors 1 and *n* are said *banal*; while all other factors are said *not banal*

Property 8

Each element of both *iMa* and *iMb*, being included between two consecutive *n* multiples one of which corresponding to n^2 (respectively between $(n-1)n$ e n^2 and between n^2 e $n(n+1)$), has a factors of its own always a number inferior to n and a number superior to n, with the peculiarity that on the whole all elements of both iMa and iMb contain, always and as a system, within their own factors inferior to n, all the iT elements, there included the banal factor 1 with the peculiarity that in one or more iMa or iMb elements may converge, at the same time, several factors inferior to n, and this causes a multiple presence of other iMa or iMb elements with the same factors inferior to n, there included the banal factor 1.

Example 13

Let's assume $n = 11$, :

iMa elements , maximum factor inferior to n

111………………………….. 3
112………………………….. 8
113………………………….. **1**
114………………………….. 6
115…………………………. 5
116…………………………. 2
117…………………………. 9
118…………………………. 4
119…………………………. 7
120 ………………………… 10

All iMa elements have a factor inferior to n = 11. All factors are present and none is repeated.

Example 14

Let's assume $n = 11$,

iMb elements maximum factor, inferior to n

122 …………………………… 4
123 …………………………… 3
124 …………………………… 2
125 …………………………… 5
126 …………………………… 6, 7, 9
127 …………………………… **1**
128 …………………………… 8
129 …………………………… 3
130 ………………………….. 10
131 ………………………….. **1**

Note that iMb elements have at least one factor inferior to n=11. All factors inferior to n=11 are present. On one iMb element (126) converge three factors inferior to n (6, 7, 9) two of wich (6 and 9) are multiples of three. The concurrence of the two multiples of three is by itself the cause of the repetition of 3 towards its natural following step (iMb 129 element). The concurrency of factor 7, in its turn, causes the repetition (two times) of the banal factor 1 (127 and 131 elements).

Example 15:
For n=101, the consecutive iMa elements are 100, excluded extremis that starting from 10.101 get to 10.200 whose major and minor n factors are shown. These factors represent all the iT elements (1, 2, 3, …, 100) none excluded, there included the banal factor 1, 12 times present. Owing to the reason that a good few factors converge, at the same time, on several iMa elements

Elements	*Factors*
10.101:	91, 39, 37
10.102:	2
10.103:	1
10.104:	24
10.105:	43
10.106:	62
10.107:	9
10.108:	28
10.109:	11
10.110:	30
10.111:	1
10.112:	64
10.113:	3
10.114:	26
10.115:	85, 35
10.116:	36
10.117:	67
10.188:	2
10.119:	3
10.120:	92, 88, 40
10.121:	29
10.122:	42
10.123:	53

10.124: 4
10.125: 81, 75, 45
10.126: 83, 61
10.127: 41, 19
10.128: 48
10.129: 7
10.130: 10
10.131: 33
10.132: 68
10.133: 1
10.134: 18
10.135: 5
10.136: 56
10.137: 93
10.138: 74
10.139: 1
10.140: 78, 65, 60, 52
10.141: 1
10.142: 22
10.143: 69, 63, 49
10.144: 32
10.145: 5
10.146: 89, 57, 38
10.147: 73
10.148: 86, 59
10.149: 51
10.150: 70, 58, 50
10.151: 1
10.152: 94, 72, 54
10.153: 71, 13
10.154: 2
10.155: 15
10.156: 4
10.157: 7
10.158: 6
10.159: 1
10.160: 80
10.161: 9
10.162: 2
10.163: 1
10.164: 84, 77, 66, 44
10.165: 95
10.166: 46, 34

10.167: 3
10.168: 82, 8
10.169: 1
10.170: 90
10.171: 7
10.172: 4
10.173: 3
10.174: 2
10.175: 55, 25
10.176: 96
10.177: 1
10.178: 14
10.179: 87, 27
10.180: 20
10.181: 1
10.182: 6
10.183: 17
10.184: 76
10.185: 97, 21
10.186: 22
10.187: 61
10.188: 12
10.189: 23
10.190: 5
10.191: 79
10.192: 98, 16
10.193: 1
10.194: 6
10.195: 5
10.196: 4
10.197: 99
10.198: 2
10.199: 47, 31
10.200: 100.

More specifically, the elements 1, 2, 3, 4, 5, 6, 7, 9, 22 besides their normal presence, reappear 31 times. At the same time, 31 factors (apparition order: 39, 37, 88, 40, 75, 45, 61, 19, 65, 60, 52, 63, 49, 57, 38, 59, 58, 50, 72, 54, 13, 77, 66, 44, 34, 8, 25, 27, 21, 16, 31) converge into posts already occupied by other factors. This phenomenon, as a matter of fact, confirms the dynamics that allow all iT elements to be present, at

least for one time, in the shape of iMa factors, with a tendency of minor factors to be repeatedly present.
Such dynamics are implicit in the propriety of both iMa and iMb for the sums of them both are always divisible by the sum of the iT correspondent

Lemma 3

In the natural scale of numbers each odd number is always followed by an even number. Each odd number consist of two odd factors. Each even number bigger than two either consists of two even factors or it is the product of an even factor and an odd one bigger than 1.

Lemma 4

The multiples of every even number, show a chronological succession becoming alternately even, odd, even, odd, to the infinity (3, 6, 9, 12, 15…; 5, 10, 15, 20, 25 …; 7, 14, 21, 28, 35 …)

Lemma 5

The multiples of every even number, being the products of at least an even factor, manifest even numbers (2, 4, 6, 8, 10 …; 4,8,12,16,20…; 6, 12, 18, 24, 30 …).

Conclusions: Three reasonings per absurd:

a) Let's assume n = 6

Let's suppose, per absurd, we don't know iMa and iMb elements and let's assume that their factors inferior to n are disposed in a manner that each element has a factors different from the banal 1, so that none of such elements is a prime number . As a consequence, we must suppose that the elements have, in order, the following factors

- 2
- 3
- 4
- 5
- 2, 3

It seems a possible logic sequence since al factors are smaller than 6 and in the last position factors 2 and 3 follow, in a correct order, their previous multiples or equals. However the above sequence is impossible for the reason that such factors belong respectively to even, odd, even, odd, even elements that contrast with property 3 which states that “when n is even the number of the elements of each set is odd, and odd elements are prevailing”. In fact, contrarily to what stated by such propriety, in this case we should have a prevalence of even elements, at an innatural distance from n. In fact, if such factors belonged to iMa elements, preceding n^2 we should face the case in wich the last element being divisible by 2 should be an even number and since n=6 if rendered squaregives an even number, we should have two natural consecutive number both even. On the other hand if the elements belonged to iMb the situation wouldn’t change since the first element, which is successive to n^2 being divisible by 2 should necessarily be an even number and consequently we should have two consecutive natural numbers as both even.

b) <u>Let’s assume n = 14</u>

Let’s suppose that also in this case, per absurd, we don’t know iMa and iMb elements and that their factors smaller than *n* are disposed in a manner in wich each element has a factor different from banal 1 and none of such elements is a prime number . In this case we therefore assume that the elements have the following factors:

- 11
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
- 11
- 12
- 13

This seems a possible logic sequence since all factors are smaller than 14; alternate disposed odd and even factors are an index of odd-even elements that alternate to one another correctly in conformity with lemma 3 dispositions; the prevalence of odd elements in the presence of even *n* ones is also correct, in conformity with property n.3. It follows that we could face the case to have either an iMa or an iMb in which no prime number may be present. On the contrary… the distance between the two factors 11 is anomalous since between the two instead of an odd distance equal to 11 does exist a distance equal to 10. In fact the second 11 should be in the place of 12 or in any case beside it. But, what is the factor we can put in the place of 11? We can't assume an even factor because we would face three consecutive evens nor we can assumne any other odd factor owing to the reason that we wouldn't respect the natural cadence of each of them (3 has its cadence in 12, 5 has its cadence in 10, 7 has no space to repeat itself as well as 9 and 13, so that banal factor 1 remains the only and unique possible perspective.

c) divisibility of iMa and iMb

If, per absurd, any multiple set a (iMa) or any multiple set b (iMb) shouldn't have in its interior any element with banal factor 1, then the biunivocal respondence with iT elements should do less of the element 1 of triangular set (iT), but this inconvenience, since number 1 is the smallest indispensable part of iT, would cause:

1) a not perfect divisibility of the elements' values sum of iMa and/or iMb by the elements' values sum of iT;
2) a not perfect divisibility of the sum of the only odd iMa and /or iMb elements' values by the sum of the only odd elements' iT values: therefore the former and the latter event would represent an imperfection within a perfect system.

In my opinion:

For any value attributed to n, each element of iT (triangular set), since its value is smaller than n, will find its identification with the factor – of equal value – of one of its multiples in at least one of the iMa elements, and iT will find also further identification with the factor – of equal value – of one of its multiples in at least one of the iMb elements.
In fact, each iT element, starting from its natural post and proceeding with an adequate step towards its own value, will get into both iMa and

iMb, where it will find in the elements of the two sets its multiples with a factor smaller than *n* and identical to iT element.

In a second time, provided an eventual capacity, iT element will find further multiples of its own with a smaller than n factor and multiples of iT element, but indeed identical to a further iT element smaller than n, or eventually it will find itself for the reason that several of its multiples have simultaneously converged into its preceding multiple with factors smaller than n, and themselves prime.

At this point it's evident that, since the number of iT, iMa, iMb elements is equal to n-1 and not banal factors are equal to n-2 (in absence of banal 1) both iMa and iMb at the end of the process will preserve at least one element which will not admit any factor for not banal element and therefore, per exclusion, such element is only and exclusively divisible by the banal factor 1 (within smaller than n factors) and only and uniquely by itself (for bigger than n factors). Moreover, since both multiple sets a and b are infinite and prime numbers are elements of such sets, we can deduce that prime numbers are infinite.

That's what I wanted to demonstrate!

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